Voltage & Current waves.

$$V(x) = \frac{1}{2}(V_r + I_r.Z_c)e^{\gamma x} + \frac{1}{2}(V_r - I_r Z_c)e^{-\gamma x}$$

γ & Z_c are complex quantities

$\gamma = \alpha + j\beta$ → phase constant measured in radian / unit length

α → attenuation constant measured in nepers / unit length

γ → Propagation Constant $= \sqrt{zy}$

Then $v(x)$ can be written as follows:

$$V(x) = A_1 e^{\alpha x} e^{j\beta x} + A_2 e^{-\alpha x} e^{-j\beta x}$$

To time Domain

$$V(t,x) = A_1 e^{\alpha x} e^{j(wt+\beta)x} + A_2 e^{-\alpha x} e^{-j(wt-\beta x)}$$

$$\therefore V(t,x) = V_1(t,x) + V_2(t,x)$$

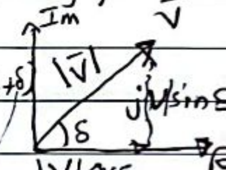
Phasors:
(represent sinusoidal quantity at fixed frequency)

EX:

$v(t) = V_{max}\cos(wt+\delta)$

maximum value

angle (phase angle)



$e^{j\alpha} = \cos\alpha + j\sin\alpha$

Remember $e^{j(wt+\beta)} = \cos(wt+\beta) + j\sin(wt+\beta)$

$$\therefore v(t,x) = \sqrt{2}\,\text{Real}\,A_1 e^{\alpha x} e^{j(wt+\beta)x} + \sqrt{2}\,\text{Real}\,A_2 e^{-\alpha x} e^{-j(wt-\beta x)}$$

$$\Rightarrow v_1(t,x) = \sqrt{2}A_1 e^{\alpha x}\cos(wt+\beta x)$$

$$v_2(t,x) = \sqrt{2}A_2 e^{-\alpha x}\cos(wt-\beta x)$$

Analysis, As x increase (distance from RE)

v_1 becomes larger i.e. $e^{\alpha x}$

v_2 " smaller i.e. $e^{-\alpha x}$

$e^{j\beta x}$ cause shift in phase of β rad/u. length & Mag = 1

⇒ First term v_1, As x increases.

\# increase in Magnitude

\# decrease in phase] incident.

Conversely from SE to RE

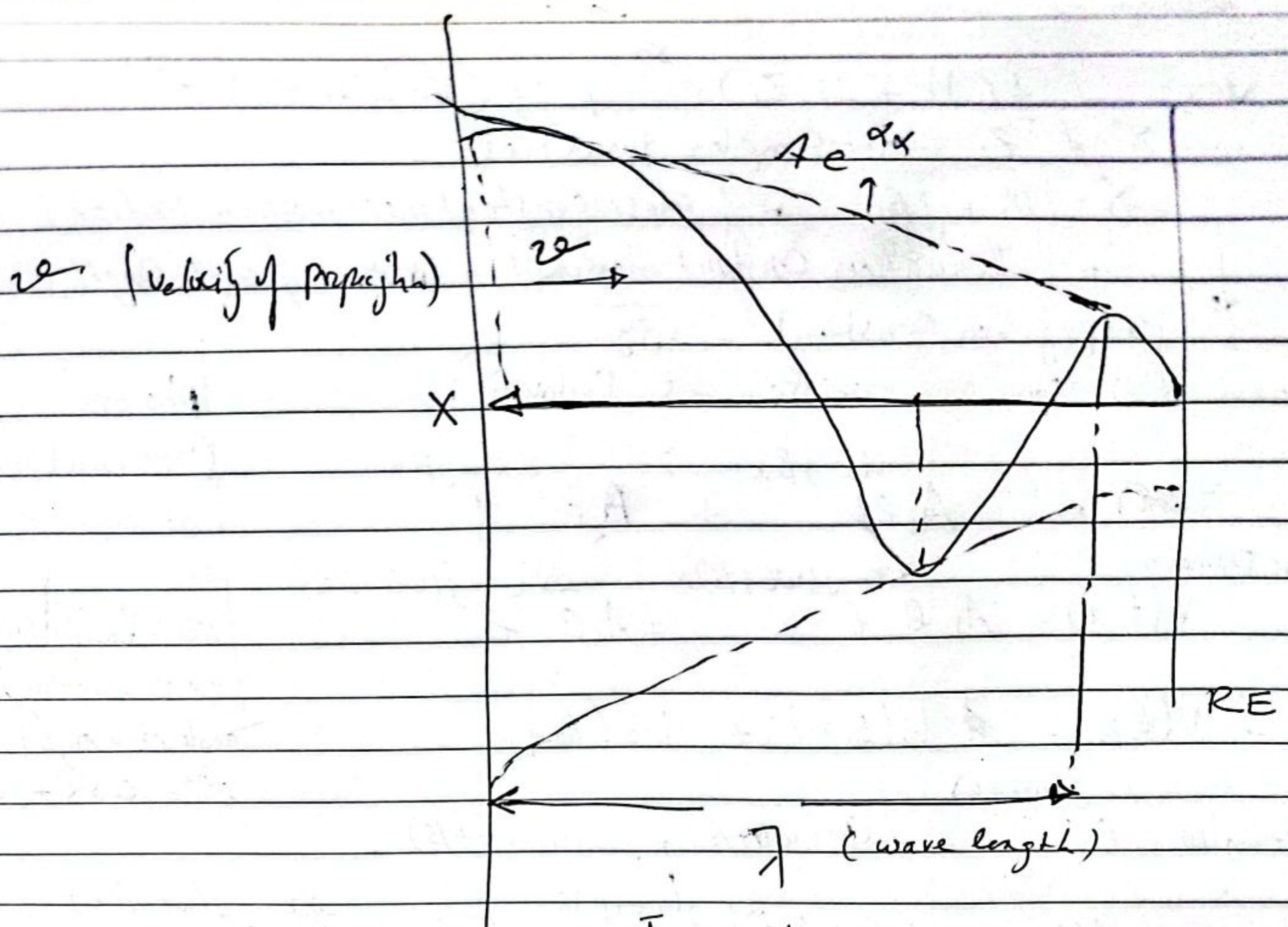
This is the characteristic of travelling waves, (incident wave) + Ref.

⇒ Second term

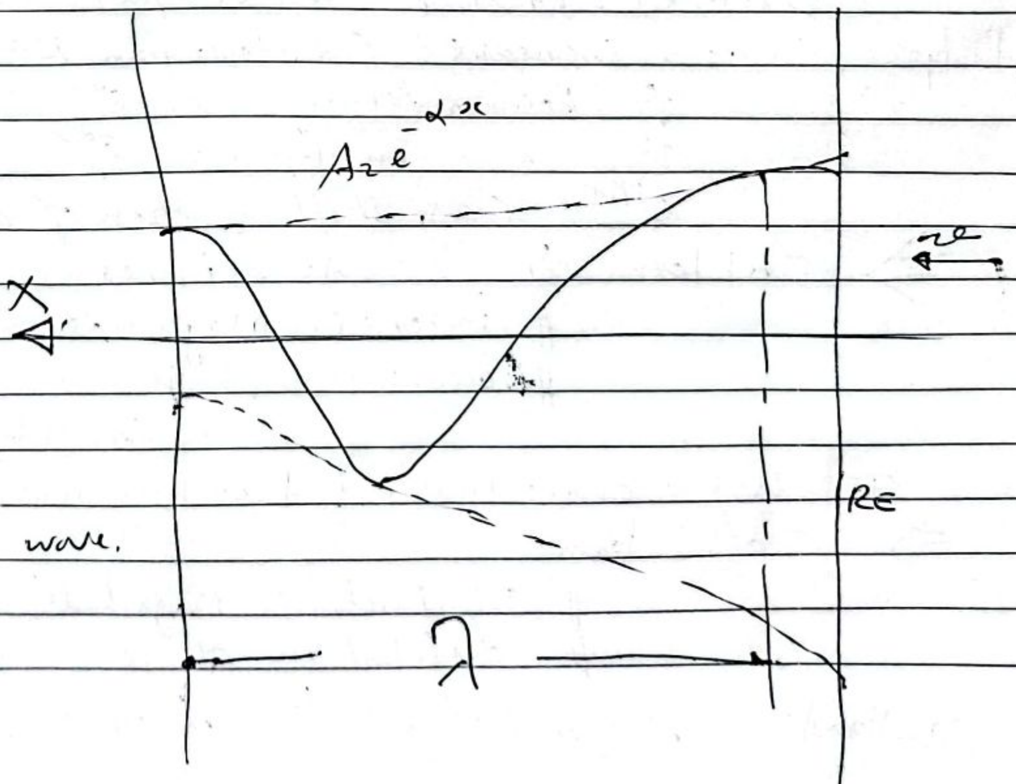
\# Diminishes in Magnitude

\# Retarded in phase] Reflected

At any point along the line the voltage is the sum of v_1 & v_2



Incident wave



Reflected wave.

Eq. $V_1(t,x)$, $V_2(t,x)$, Peak magnitude requires $(\omega t-\beta x)=2k\pi$

$$\Rightarrow x=\frac{\omega}{\beta}t-\frac{2k\pi}{\beta}$$

$$\Rightarrow \frac{dx}{dt}=\frac{\omega}{\beta} \Rightarrow \text{velocity of Propagation}$$

$$\Rightarrow v=\frac{\omega}{\beta}=\frac{2\pi f}{\beta}$$

$\therefore$ wavelength λ. (the distance x on the wave at which phase shift $=2\pi$)

$$\beta\lambda=2\pi \Rightarrow \lambda=\frac{2\pi}{\beta}$$

(km OR Miles) depends on β

if losses of the line are neglected i.e $G=\cancel{R}=0$

$$\Rightarrow \alpha=0$$

$$\therefore \beta=\omega\sqrt{LC} \quad \& \quad Z_c=\sqrt{L/C} \text{ (pure resistive)}$$

$$\therefore v=\frac{\omega}{\beta}=\frac{\cancel{\omega}}{\cancel{\omega}\sqrt{LC}}=\frac{1}{\sqrt{LC}}$$

$$\& \ \lambda=\frac{1}{f\sqrt{LC}} \ m$$

For lossless line, the velocity of Propagation $v=\dfrac{1}{\sqrt{LC}}$ m/s

$\mu_0=8.85\times10^{-12}$ F/m

$k_0=4\pi\times10^{-7}$ H/m

$$=\frac{1}{\sqrt{\frac{\mu_0}{\cancel{2\pi}}\cancel{\ln\frac{D}{r}}\times\frac{\cancel{2\pi}k_0}{\cancel{\ln\frac{D}{r}}}}}$$

neglecting ~~internal~~ the inductance due to internal flux i.e $r=r'$

$$\approx \frac{1}{\sqrt{\mu_0 k_0}} \Rightarrow \text{velocity of light}=3\times10^8 \text{ m/sec}$$

$$\& \ \lambda=\frac{1}{f\sqrt{LC}} \approx \begin{array}{l}6000\text{ km for }50\text{ HZ}\\ 5000\text{ km for }60\text{ HZ}\end{array}$$

Characteristic impedance $Z_c=\sqrt{L/C}$

$$\approx \frac{1}{2\pi}\sqrt{\mu_0/k_0}\ \ln\frac{GMD}{GMR_c}$$

$\approx$ between 250 _ 400 Ω

Dr. Hamid

Surge Impedance loading

if the load is = Z_c Lossless line Z_c is resistive

$\Rightarrow I_r = \dfrac{V_r}{Z_c}$

Surge impedance loading SIL $\Rightarrow$ The Power delivered by line to a purley resistive load = to the line surge impedance.

$$SIL = \frac{(KV_L \text{ rated})^2}{Z_c} \; MW$$

Now, for lossless line

$$\gamma = j\beta \Rightarrow \cosh\gamma x = \cosh j\beta x = \cos\beta x$$
$$\sinh\gamma x = \sinh j\beta x = j\sin\beta x$$

$\therefore V(x) = V_r\cos\beta x + j\, I_r Z_c \sin\beta x$

$I(x) = j\dfrac{V_r}{Z_c}\sin\beta x + I_r\cos\beta x$

But at SE $x = l$

$\therefore V_s = V_r\cos\beta l + j\, I_r Z_c \sin\beta l$

$I_s = j\dfrac{V_r}{Z_c}\sin\beta l + I_r\cos\beta l$

For lossless line

$\beta = j\chi$

$\alpha = 0$

$\beta = 90^\circ$

$A = \cos\beta l$

$\gamma = j\beta$

Now Substitute by $I_r = \dfrac{V_r}{Z_c}$ (load = Z_c)

$V_s = V_r(\cos\beta l + j\sin\beta l) \Rightarrow V_s = V_r \angle \beta x$

$I_s = I_r(\cos\beta l + j\sin\beta l) \Rightarrow I_s = I_r \angle \beta l$

Since Z_c is resistive (No reactive Component) $\Rightarrow$ No Q from the source
$\Rightarrow$ at SIL loading the reactive losses in the line inductance are exactlly offset by the reactive power Supplied by the shunt admittance.

SIL varies from 150MW for 220kV lines to 2000MW for 765 kV